F = S (1 + r) – D

= ₹ 220 (1.0375) – 2.5

= ₹ 225.75

(2) Arbitrage

(i) Action: Since future is overpriced. Hence, buy spot & sell future.

(ii) Process:

Today:

- Borrow ₹ 220 @ 15% p.a. for 3 months & buy spot
- Sell future @ 230

After 3 Months

Cash Flows =	Sell Stock	= +230
	Dividend	= +2.50
	Repay 220(1.0375)	= −228.25
	Arbitrage Gain	= 4.25

Question – 20

The share of X Ltd. is currently selling for ₹ 300. Risk free interest rate is 0.8% per month. A three month futures contract is selling for ₹ 312. Develop an arbitrage strategy and show what your riskless profit will be 3 months hence assuming that X Ltd. will not pay any dividend in the next three months.

(SM TYK – 06)

Solution:

(I) Theoretical Future Price

F = S (1 + r)

= 300 × $(1.008)^3$

= ₹ 307.26

(II) Arbitrage

(i) Action: Since future is overpriced, hence buy spot & sell future.

(ii) Process

Today

- Borrow ₹ 300 @ 0.8% p.m. & buy share
- Sell future at ₹ 312

After 3 Months

Sell Stock	= + 312
Repay $300(1.008)^3$	= 307.27
Arbitrage Gain	= 4.74

Note: यदि rate per month दिया है तो monthly compounding लेना है।

Question – 21

The 6-months forward price of a security is ₹ 208.18. The borrowing rate is 8% per annum payable with monthly rests. What should be the spot price?

(SM TYK – 01)

Solution:

Forward Price $= S\,(1 + r)^n$

208.18 $= S\,(1.0067)^6$

S = ₹ 200

Question – 22

On 31-8-2011, the value of stock index was ₹ 2,200. The risk free rate of return has been 8% per annum. The dividend yield on this Stock Index is as under:

Month	Dividend Paid p.a.
January	3%
February	4%
March	3%
April	3%
May	4%
June	3%

July	3%
August	4%
September	3%
October	3%
November	4%
December	3%

Assuming that interest is continuously compounded daily, find out the future price of contract deliverable on 31-12-2011. Given: $e^{0.01583} = 1.01593$

(SM TYK – 03)

Solution:

Average Dividend Yield

$$= \frac{3 + 3 + 4 + 3}{4}$$

= 3.25% p.a.

$$F = S \times e^{(r-d)t}$$

$$= 2{,}200 \times e^{(0.08 - 0.0325)4/12}$$

$$= 2{,}200 \times e^{0.01583}$$

$$= 2{,}200 \times 1.01593$$

$$= 2{,}235.046$$

Question – 23

The NSE-50 Index futures are traded with rupee value being ₹100 per index point. On 15th September, the index closed at 1195, and December futures (last trading day December 15) were trading at 1225. The historical dividend yield on the index has been 3% per annum and the borrowing rate was 9.5% per annum.

(i) Determine whether on September 15, the December futures were under-priced or overpriced?

(ii) What arbitrage transaction is possible to gain out this mispricing?

(iii) Calculate the gains and losses if the index on 15^{th} December closes at (a) 1260 (b) 1175.

Assume 365 days in a year for your calculations

(Exam November – 2019) (8 Marks)

Solution:

(I) Theoretical Future Price

Future Price = $S \times [1 + (r - d)t]$

$= 1{,}195 \left[1 + (0.095 - 0.03) \times \frac{91}{365}\right]$

= 1,214.37

Future Price = 1,214.37 × 100 = 1,21,437

Actual Future Price = 1,225 × 100 = 1,22,500

(II) Action

(i) Since actual future price is more than Theoretical Future Price hence future is overpriced.

(ii) Since future is overpriced, hence buy spot & sell future [short position]

(III) Arbitrage Process

- Borrow ₹ (1,195 × 100) @ 9.5% p.a. for 91 days & buy index

- Sell future (short position) at 1,225 × 100

After 91 Days

	1260	1175
Sell Index	(1,260 × 100) + 1,26,000	(1,175 × 100) + 1,17,500
Dividend Received $\left[1{,}195 \times 3\% \times \frac{91}{365}\right] \times 100$	+ 893.79	+ 893.79
Repayment $1{,}195 \left[1 + \left(0.095 \times \frac{91}{365}\right)\right] \times 100$	- 1,22,330.35	- 1,22,330.35

Gain/Loss on short position	(-35 × 100) = -3,500	(+50 × 100) = +5,000
Arbitrage	1,063.44	1,063.44

Question – 24

Suppose current price of an index is ₹ 13,800 and yield on index is 4.8% (p.a.). A 6 months future contract on index is trading at ₹ 14,340.

Assuming that risk free rate of interest is 12%. Show Mr. X (an arbitrageur) can earn an abnormal rate of return irrespective of outcome after 6 months . You can assume that after 6 months index closes at ₹ 10,200 and ₹ 15,600 and 50% of stock included in index shall pay dividend in next 6 months. Also Calculate implied risk free rate.

Solution:

Theoretical Future Price

F = 13,800(1.06)

= 13,800 × 50% × 4.8%

= 14,628 – 331.20

= 14,296.80

Since future is overpriced hence buy spot & sell future (short position)

After 6 Months **14340 ↓**

	10,200	15,600
Sell Index	+ 10,200	+ 15,600
Dividend	+ 331.20	+ 331.20
Buy Index (Spot)	- 13,800	- 13,800
Gain/Loss on Short Position	+ 4,140	- 1,260
Arbitrage Gain	871.20	871.20

Implied Risk Free Rate $= \frac{871.20}{13,800} \times 100 \times \frac{12}{6}$

= 12.63% p.a.

Note: जब भी Implied Risk Free Rate मांगेगा Borrowing नहीं लेना है।

Question – 25

A future contract is available on R Ltd. that pays an annual dividend of ₹4 and whose stock is currently priced at ₹125. Each future contract calls for delivery of 1,000 shares to stock in one year, daily marking to market. The corporate treasury bill rate is 8%.

Required:

(i) Given the above information, what should the price of one future contract be?

(ii) If the company stock price decreases by 6%, what will be the price of one futures contract?

(iii) As a result of the company stock price decrease, will an investor that has a long position in one futures contract of R Ltd. realizes a gain or loss ? What will be the amount of his gain or loss?

(Ignore margin and taxation, if any)

(Exam Nov – 2019) (6 Marks)

Solution:

(i) Price of one future contract

F = S (1 + r) – D

= ₹ 125 (1.08) – 4

= ₹ 131

= 131 × 1,000 shares

= ₹ 1,31,000

(ii) If stock price decrease by 6%

S = 125 × 0.94 = 117.50

F = 117.50 (1.08) – 4

= ₹ 122.90

= ₹ 122.90 × 1,000 shares

= ₹ 1,22,900

(iii) Gain or Loss on Long Position

If stock price decrease then loss on long position

(1,31,000 – 1,22,900) = 8,100 (Loss)

Question – 26

The price of ACC stock on 31 December 2010 was ₹ 220 and the futures price on the same stock on the same date, i.e., 31 December 2010 for March 2011 was ₹ 230. Other features of the contract and related information are as follows:

Time to expiration	- 3 months (0.25 year)
Borrowing rate	- 15% p.a.
Annual dividend on the stock	- 25% payable before 31.03.2011
Face Value of the Stock	- ₹ 10

i. Based on the above information, what should be the futures price?

ii. Show the process of arbitrage

Solution:

i. Futures price = 220 + (220 × 0.15 × 0.25) – (0.25 × 10) = 225.75

ii. He will buy the ACC stock at ₹ 220 by borrowing the amount @ 15 % for a period of 3 months and at the same time sell the March 2011 futures on ACC stock. By 31st March 2011, he will receive the dividend of ₹ 2.50 per share. On the expiry date of 31st March, he will deliver the ACC stock against the March futures contract sales.

The arbitrager's inflows/outflows are as follows:

Sale proceeds of March 2011 futures	₹ 230.00
Dividend	₹ 2.50
Total (A)	**₹ 232.50**
Pays back the Bank	₹ 220.00
Cost of borrowing	₹ 8.25
Total (B)	**₹ 228.25**
Balance (A) – (B)	**₹ 4.25**

Thus, the arbitrage earns ₹ 4.25 per share without involving any risk.

(VII) BETA MANAGEMENT

Question – 27

On April 1, 2015, an investor has a portfolio consisting of eight securities as shown below:

Security	Market Price	No. of Shares	Value
A	29.40	400	0.59
B	318.70	800	1.32
C	660.20	150	0.87
D	5.20	300	0.35
E	281.90	400	1.16
F	275.40	750	1.24
G	514.60	300	1.05
H	170.50	900	0.76

The cost of capital for the investor is 20% p.a. continuously compounded. The investor fears a fall in the prices of the shares in the near future. Accordingly, he approaches you for the advice to protect the interest of his portfolio.

You can make use of the following information:

(1) The current NIFTY value is 8500.

(2) NIFTY futures can be traded in units of 25 only.

(3) Futures for May are currently quoted at 8700 and Futures for June are being quoted at 8850.

You are required to calculate:

(i) The beta of his portfolio.

(ii) The theoretical value of the futures contract for contracts expiring in May and June. Given ($e^{0.03}$ =1.03045, $e^{0.04}$ = 1.04081, $e^{0.05}$ =1.05127)

(iii) The number of NIFTY contracts that he would have to sell if he desires to hedge until June in each of the following cases:

(A) His total portfolio

(B) 50% of his portfolio

(C) 120% of his portfolio

(SM TYK – 13)

Solution:

(i) Calculation of Beta of Portfolio

Stocks	Market Price	No. of Shares	Value	Weight	Beta	W × B
A	29.40	400	11,760	0.01182	0.59	0.0070
B	318.70	800	2,54,960	0.2564	1.32	0.3384
C	660.20	150	99,030	0.0996	0.87	0.0866
D	5.20	300	1,560	0.00157	0.35	0.0005
E	281.90	400	1,12,760	0.1134	1.16	0.1315
F	275.40	750	2,06,550	0.2077	1.24	0.2575
G	514.60	300	1,54,380	0.15524	1.05	0.1630
H	170.50	900	1,53,450	0.1543	0.76	0.1173
			9,94,450		Bp = 1.102	

(ii) Calculation of theoretical future price

May future (2 months)

F $= Se^{rt}$

$= 8{,}500 \times e^{0.20 \times 2/12}$

$= 8{,}500 \times e^{0.0333}$

$= 8{,}500 \times 1.03387$

$= 8{,}787.89$

Interpolation

$e^{0.03}$ ------------------------ 1.03045

$e^{0.0333}$

$e^{0.04}$ ------------------------ 1.04081

$e^{0.01}$ 0.01036

$$1.03045 + \frac{0.01036}{0.01} \times 0.0033$$

$= 1.03387$

June Future

F $= 8,500 \times e^{(0.20 \times 3/12)}$

$= 8,500 \times 1.05127$

= ₹ 8,935.79

(iii) Calculation of no. of contacts

$$\text{No. of Contract} = \frac{V_P \times (B_T - B_P)}{F \times M}$$

(A) Total portfolio

$$\text{No of contracts} = \frac{9,94,450 \times (0 - 1.102)}{8,850 \times 25}$$

= 4.95 Contracts

5 contracts sold

(B) 50% of Portfolio

$$\text{No of contracts} = \frac{9,94,450 \times 50\% \times (0 - 1.102)}{8850 \times 25}$$

= 2.48 Contracts

2 contracts sold or 3 contracts

(C) 120% of Portfolio

$$\text{No of contracts} = \frac{(9,94,450 \times 120\%) \times (0 - 1.102)}{8850 \times 25}$$

= 5.94 Contracts

6 contracts sold.

Question – 28

Details about portfolio of shares of an investor is as below:

Shares	No. of shares (lakh)	Price per share	Beta
A Ltd.	3.00	₹ 500	1.40
B Ltd.	4.00	₹ 750	1.20
C Ltd.	2.00	₹ 250	1.60

The investor thinks that the risk of portfolio is very high and wants to reduce the portfolio beta to 0.91. He is considering two below mentioned alternative strategies:

(i) Dispose off a part of his existing portfolio to acquire risk free securities, or

(ii) Take appropriate position on Nifty Futures which are currently traded at 8125 and each Nifty points is worth ₹ 200.

You are required to determine:

(1) Portfolio beta,

(2) The value of risk free securities to be acquired,

(3) The number of shares of each company to be disposed off,

(4) The number of Nifty contracts to be bought/sold; and

(5) The value of portfolio beta for 2% rise in Nifty.

(SM TYK – 12)

Solution:

1. Beta of Portfolio

Shares	MPS	No.	Amount	Weights	Beta	β × W
A	3.00	500	1,500	0.3	1.40	0.42
B	4.00	750	3,000	0.6	1.20	0.72
C	2.00	250	500	0.1	1.60	0.16
		Vp =	5,000		Bp =	1.30

2. Value of Risk Free Securities

$$0.91 = \frac{(5{,}000 \times 1.30) - (x \times 1.30) + (x \times 0)}{5{,}000}$$

$$x = 1{,}500$$

ICAI

$$Wp = \frac{B_T}{B_P} = \frac{0.91}{1.30} = 0.70$$

Rf = 5,000 × 0.3 = 1,500

Investment in Rf = ₹ 1,500

3. No. of Shares

Shares	Weights	Amount (Lakh)	MPS	No. of Shares in (Lakh)
A	0.3	450	500	0.9
B	0.6	900	750	1.20
C	0.1	150	250	0.6
		1,500		

4. No. of Nifty Contracts

$$\text{No.} = \frac{V_P \times (B_T - B_P)}{F \times M}$$

$$= \frac{5{,}000 \times (0.91 - 1.30)}{8{,}125 \times 200}$$

= 120 Contracts Short

5. Value of Portfolio Beta

Portfolio (Bp = 1.30)
₹ 5,000 Lakhs

↓

Portfolio will rise by (2 × 1.30)
= 2.6%

↓

Gain on Long Position
(5,000 × 2.6%) = +130

Nifty 5,000(0.91 – 1.30)
1,950 Lakhs Short Position

↓

Nifty rise by 2%

↓

Loss on Short Position of Nifty
1,950 × 2% = -39

Overall Gain OR Loss = +91 Lakhs

% of Gain/Loss $= \frac{91}{5,000} \times 100 = 1.82\%$

Beta $= \frac{\Delta \text{ In Portfolio Return}}{\Delta \text{ In Market Rate}} = \frac{1.82}{2} = 0.91$

Question – 29

On January 1, 2013 an investor has a portfolio of 5 shares as given below:

Security	Price	No. of Shares	Beta
A	349.30	5,000	1.15
B	480.50	7,000	0.40
C	593.52	8,000	0.90
D	734.70	10,000	0.95
E	824.85	2,000	0.85

The cost of capital to the investor is 10.5% per annum.

You are required to calculate:

(i) The beta of his portfolio.

(ii) The theoretical value of the NIFTY futures for February 2013.

(iii) The number of contracts of NIFTY the investor needs to sell to get a full hedge until February for his portfolio if the current value of NIFTY is 5900 and NIFTY futures have a minimum trade lot requirement of 200 units. Assume that the futures are trading at their fair value.

(iv) The number of future contracts the investor should trade if he desires to reduce the beta of his portfolios to 0.6.

No. of days in a year be treated as 365.

Given: In (1.105) = 0.0998 and $e^{(0.015858)} = 1.01598$

(SM TYK – 11)

Solution:

1. Beta of Portfolio

Shares	MPS	No.	Amount	Weights	Beta	β × W
A	349.30	5,000	17,46,500	0.093	1.15	0.107
B	480.50	7,000	33,63,500	0.178	0.40	0.071

C D E	593.52 734.70 824.85	8,000 10,000 2,000	47,48,160 73,47,000 16,49,700	0.252 0.390 0.087	0.90 0.95 0.85	0.227 0.370 0.074
		Vp =	1,88,54,860		Bp =	0.849

2. Theoretical Future

$F = S \times e^{rt}$

$= 5{,}900 \times e^{0.105 \times 58/365}$

$= 5{,}900 \times e^{0.01668}$

$= 5{,}900 \times 1.01682$

$= 5{,}999.24$

3. No. of Contracts (B_T = 0)

$$\text{No.} = \frac{V_P \times (B_T - B_P)}{F \times M}$$

$$= \frac{1{,}88{,}54{,}860 \times (0 - 0.849)}{5{,}999.24 \times 200}$$

= 13.35 or 13 Contracts Short

4. B_T = 0.6

$$\text{No.} = \frac{1{,}88{,}54{,}860 \times (0.6 - 0.849)}{5{,}999.24 \times 200}$$

= 3.91 or 4 Contracts Short

Question – 30

Following information is available for consideration:

BSE Index	25,000
Value of portfolio	₹ 50,50,000
Risk free interest rate	9% p.a.
Dividend yield on Index	6% p.a.

Beta of portfolio 1.5

We assume that a future contract on the BSE index with 4 months maturity is used to hedge the value of portfolio over next 3 months. One future contract is for delivery of 50 times the index.

Based on the above information calculate:

(i) Price of future contract.

(ii) Gain on short futures position if index turns out to be 22,500 in 3 months.

Note: Daily compounding (exponential) formula is not required to be used.

(RTP May – 2022, Exam July – 2021) (8 Marks)

Solution:

(i) Price of Future Contracts

$$F = S\,[1 + (r - d)t]$$

$$= 25{,}000\,[1 + (0.09 - 0.06)4/12]$$

$$= ₹\ 25{,}250$$

Price of 1 Future Contract = ₹ 25,250 × 50

= ₹ 12,62,500

No. of Contracts $= \dfrac{V_p(B_T - B_p)}{F \times M}$

$= \dfrac{50{,}50{,}000 \times (0 - 1.5)}{25{,}250 \times 50}$

= 6 Contracts Short

(ii) Gain or Loss on Short Position

3 months spot = 22,500

[हमने Contract 4 month future पर किया है तो, 3 month के End पर काटना है Spot पर नहीं कटेगा, ये Contract 1 month future पर सौदा कटेगा|]

1 month future at the end of 3 months

F $= 22{,}500\left[1 + (0.09 - 0.06)\frac{1}{12}\right]$

$= 22{,}556.25$

Gain on Short Position $= (25{,}250 - 22{,}556.25) \times 50 \times 6$

$=$ ₹ 8,08,125

Question – 31

A Future contract on BSE Index with 4 months maturity is used to hedge the value of the portfolio over the next 3 months. One future contract for delivery is 50 times of the index.

The following information is available :

Value of the portfolio	₹ 1,16,00,000
BSE Sensex on 1st January 2022	58,580
(Anticipated on 1st September 2021)	
BSE Sensex on 1st January 2022	56641.25
(Anticipated on 1st December 2021)	
Dividend Yield of Index	6% p.a
181 day's treasury bills offers a rate of interest	9% p.a.
Beta of the portfolio	1.5

You are required to calculate

(i) The present value of the Sensex as on 1st September 2021

(ii) Turned out value of the Sensex on 1st December 2021

(iii) The number of contracts to hedge the portfolio.

(Exam December – 2021) (8 Marks)

Solution:

(I) Spot (01/09/2021)

F = S [1 + (r – d)t]

₹ 58,580 = S [1 + (0.09 – 0.06)4/12]

S = ₹ 58,000

(II) Spot Rate (01/12/2021)

56,641.25 = S [1 + (0.09 – 0.06) × 1/12]

S = ₹ 56,500

(III) No. of Contracts

$$\text{No. of Contracts} = \frac{V_P(B_T - B_P)}{F \times M}$$

$$= \frac{1,16,00,000\ (0 - 1.50)}{58,580 \times 50}$$

= 6 Contracts Short

Question – 32

Mr. X is having a portfolio of shares worth ₹ 170 lakhs at current price and cash ₹ 30 lakhs. The beta of share portfolio is 1.6. After 3 months the price of shares dropped by 3.2%.

Determine:

(i) Current portfolio beta.

(ii) Portfolio beta after 3 months if Mr. X on current date goes for long position on ₹ 200 lakhs Nifty futures.

(Exam July – 2021) (8 Marks)

Solution:

(I) Beta of Portfolio

$$B_P = \frac{(170 \times 1.60) + (30 \times 0)}{200}$$

= 1.36

(II) Beta After 3 Months

Price of shares dropped by 3.2%, It means Nifty dropped by

$$\beta = \frac{\Delta \text{ Stock}}{\Delta \text{ Market}} = 1.6 = \frac{3.2}{\Delta \text{ Market}} = 2\%$$

Gain or Loss

Portfolio (Long)
Share = 170 Lakhs
Cash = 30 Lakhs
↓
Share price down 3.2%
↓
Loss = (170 × 3.2%) = 5.44

Nifty
200 Lakhs (Long Position)
↓
Nifty down by 2%
↓
200 × 2% = 4

Overall Gain or Loss = 5.44 + 4 = 9.44

$$\text{Loss in (\%)} = \frac{9.44}{200} \times 100 = 4.72\%$$

$$\text{Beta} = \frac{4.72\%}{2\%} = 2.36$$

Question – 33

Mr. SG sold five 4-Month Nifty Futures on 1st February 2020 for ₹ 9,00,000. At the time of closing of trading on the last Thursday of May 2020 (expiry), Index turned out to be 2100. The contract multiplier is 75.
Based on the above information calculate:

(i) The price of one Future Contract on 1st February 2020.

(ii) Approximate Nifty Sensex on 1st February 2020 if the Price of Future Contract on same date was theoretically correct. On the same day Risk Free Rate of Interest and Dividend Yield on Index was 9% and 6% p.a. respectively.

(iii) The maximum Contango/Backwardation.

(iv) The pay-off of the transaction.

Note: Carry out calculation on month basis.

(RTP November – 2020)

Solution:

(i) Price of One Future Contract

$$\text{Price} = \frac{₹\,9,00,000}{5} = ₹\ 1,80,000$$

$$F = \frac{₹\ 1,80,000}{75} = 2,400$$

(ii) Spot Price

$F = S\,[1 + (r - d)t]$

$2,400 = S\,[1 + (0.09 - 0.06)4/12]$

$S = 2,376$

(iii) Contango/Backwardation

Generally, Future is more than Spot, if Future is more than Spot it is called "सीधा बदला" [Contango]. If S > F, उल्टा बदला [Backwardation]

Basis = S – F   Negative ⟶ Contango

Positive ⟶ Backwardation

Maximum Contango = (2,376 – 2,400)

= ₹ 24

(iv) Gain or Loss

Gain on Short Position = (2,400 – 2,100) × 75 × 5

= ₹ 1,12,500